1. A local company advertises on the radio and in the newspaper. Let x and y represent the amounts (in thousands of dollars) spent on radio and newspaper advertising, respectively. The company's profit based on this advertising has been determinant to be (in thousands of dollars)

$$P(x,y) = -2x^2 - xy - y^2 + 8x + 9y + 10$$

Determine the amount of money this company should spent on each type of advertising in order to maximize its profit.

2. A company makes two kinds of personal computers. One sells for $1200 and the other sells for $1500. If the company sells x of $1200 computers and y of $1500 computers, the revenue (in hundreds of dollars) is $R = 12x + 15y$. Suppose that the cost of making the computers is known to be $C = x^2 - xy + y^2$ hundred dollars. Determine how many of each kind of computer should be made in order to maximize the profit made by the company.

3. A manufacturer makes x thousand radios and y thousand tape recorders. The resulting revenue (in thousands of dollars) is $R(x,y) = -2x^2 + xy - y^2 + 30x - 4y + 20$. How many of each product should be made in order to have maximum revenue? Also, yow much is that maximum revenue?

4. Find three positive numbers that satisfy both of these conditions:

(i) Their sum is 27

(ii) The sum of the squares of the numbers is as small as possible

5. A farmers cooperative plans to construct a rectangular storage facility for grain. The building will be made to contain a volume of 16,000 cubic feet. The cost for the roof and the floor is $10 for per square foot, the cost for the sides is $5 for square foot.

(i) What should be the dimensions of the storage facility in order to minimize the cost of materials used?

(ii) What is the cost of materials for a storage facility built to the specifications determined in part (i)?

Maximum and minimum on a constraint

Some applied problems require that we maximize or minimize a function subject to some additional condition or constraint. Sometimes we can solve the problem by inserting the constraint information into the function to be maximized or minimized - either by immediate substitution or by manipulating a constraint equation and then substituting. Unfortunately, such equation manipulation cannot always be done easily, and sometimes it can produce en expression that is difficult to work with. The French-Italian mathematician Joseph Louis Lagrange developed the method that uses partial derivatives to solve such constrained maximum/minimum problems.

The method of Lagrange Multipliers

The relative extrema of $z = f(x, y)$ subject to t he constraint $g(x, y) = 0$ are found from the new function $F(x, y) = f(x, y) + \lambda g(x, y)$

by solving the system $\begin{cases} F_x(x, y, \lambda) = 0 \\ Fy(x, y, \lambda) = 0 \\ F_\lambda(x, y, \lambda) = 0 \end{cases}$ for λ and then for points (x, y). The relative extrema are included among the solution points (x, y) obtained.

Examples

1. Determine minimum value of $f(x, y) = 2x^2 + y^2 + 7$ subject to the constraint $x + y = 18$.
2. Determine the points of the relative extrema of the function $f(x, y) = 2x + 3y$ subject to the constraint $4x^2 + 2y^2 = 9$. Find the values of these extrema.
3. Determine the minimum cost of producing 24,000 units of a product, assuming the cost is $20x + 60y$ and the production function is $f(x, y) = 200x^{1/2}y^{3/4}$, where x is the number of units of labour and y is a number of units of capital.